Mathematical Statistics Qualifier Examination
Part I of the STAT AREA EXAM
May 24, 2023; 9:00 AM - 11:00 AM

There are 4 problems. You are required to solve them all. Show detailed work for full credit.

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

NAME: ______________________________ **ID:** __________________________

Signature: ______________________________

1. Let $X_1, \cdots, X_n$ be a random sample from a Uniform$(0, \theta)$ distribution.
   Let $Y_n = \max\{X_1, \cdots, X_n\}$.

   (a) Find the cdf of Y_n.

   (b) Find the limiting distribution of $Z_n = n(\theta - Y_n)$.

   (c) What is the name of the limiting distribution obtained in part (b)?

2. Let X be a random variable such that $P(X \leq 0) = 0$ and let $\mu = \mathrm{E}(X)$ exist. Show that $P(X \geq 2\mu) \leq 1/2$.

3. Let $X_1, \cdots, X_n$ be a random sample from an exponential distribution with pdf

$$f(x) = e^{-(x-\theta)}, \; x \geq \theta, \; -\infty < \theta < \infty.$$

   Find the UMVUE of θ.

4. Let $X_1, \cdots, X_n$ be a random sample from the beta distribution with $\alpha = \beta = \theta$ and $\Omega = \{\theta\colon \theta = 1, 2\}$. Find the likelihood ratio test statistic for testing $H_0\colon \theta = 1$ vs. $H_1\colon \theta = 2$.